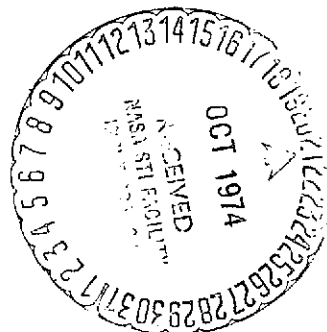


PROBLEMS OF CALCULATING THE THERMAL RADIATION OF MULTI-LAYERED STRUCTURES OF THE ICE AND SNOW TYPES BY MEANS OF ORIENTED GRAPHS USING AN ELECTRONIC COMPUTER

M. D. Rayev, Ye. A. Sharkov, T. A. Shirayeva, and  
V. S. Etkin

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AUTHORS' ABSTRACT

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M. D. Rayev, Ye. A. Sharkov, T. A. Shiryayeva, and V. S. Etkin

INTRODUCTION

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The radiothermal radiation of natural objects depends on a large number of factors, including the shape of the surface, the parameters of the bounding media, the presence of multiple reflections, bulk scattering, and so on. When radiothermal radiation of such objects is analyzed, various approximate models are used. The simplest models include the model in the form of a homogeneous radiating semi-space, the model of a layered medium with smooth boundaries, the model of a radiating semi-space with rough boundaries, the model of plane layered media with bulk scattering, and others [1].

The radiobrightness temperature ( $T_{br}$ ), which is related to the thermodynamic temperature ( $T_0$ ) by the relation:

$$T_{br}(\lambda) = \epsilon(\lambda) \cdot T_0 \quad (1)$$

where  $\epsilon(\lambda)$  is the emissivity of the material of the surface layer at a given wavelength, is a quantitative characteristic of radiothermal radiation. The emissivity of a material depends on the coefficient of reflection  $\rho$  of the material in the surface layer and the law of the distribution of inhomogeneities for the surface section under consideration:

$$\epsilon(\lambda) = 1 - \rho(\lambda).$$

The coefficient of reflection of a material  $\rho(\lambda)$  is determined by the electromagnetic characteristics of the material ( $\epsilon$  and  $\text{tg } \delta$ ), the angle of incidence, polarization, and the structure of the object under study.

The present study examines the application of the method of oriented graphs to the analysis of complex structures with well-defined plane boundaries, whose roughness satisfies Rayleigh's criterion. In natural conditions, this idealization can be satisfied by snow-ice-water structures, namely: ice at a water surface, ice with snow at a water surface, thawing water on the surface of ice, and other analogous structures. //4

After a description of the method, in this report an analysis is made of several particular cases of structures of the snow-ice-water type and the effectiveness of this method is shown with their examples.

### 1. Application of the Method of Oriented Graphs to Calculating Multilayered Structures

Let us examine a multilayered structure of  $n$  plane layers with dielectric permittivities  $\epsilon_n$  and thicknesses  $z_n$ . This problem can be solved by the direct solution of Maxwell's equations with multiple boundary conditions. However, the solution of this problem can be essentially simplified by using oriented graphs, which permit graphically representing the effects of multiple reflection and calculating the multilayered structures with the aid of an electronic computer in a quite economical fashion.

In radiotechnology, the method of graphs is widely used in analyzing complex parametric networks of various types and UHF devices [2,3].

In optics, multilayered structures in which the propagation of waves can be considered as a process of the transformation of space frequencies is an analog of linear systems transforming frequencies.

The passage of a signal through such a structure can be described with a system of equations that can be represented and solved by the method of graphs. This representation of the passage of a signal through a layered structure with the aid of graphs

permits graphically representing the transformation of signals, tracing all feedback loops, and calculating the response at any point in the system.

An oriented graph is a set of nodes and branches with a direction [3]. Each branch is characterized by a quantity called the branch transfer and denotes those operations which are done in the system on the signal. Each node stands for the signal at a given point in the system. /5

For simplicity let us consider the graph of the path of a wave through the interface of two media (Fig. 1). The wave striking the interface will be partially reflected and will partially pass through the interface. A plane wave impinging on the surface at the angle  $\theta_1$  to its normal has the space frequency  $K_1 \sin \theta_1$ , where  $K_1$  is the wave number of the impinging wave ( $K = 2\pi / \sqrt{\epsilon_n}$ ). In the language of frequency, when a wave is reflected a transformation occurs in the space frequency (change in direction and phase of the incident wave) by the value of the doubled space frequency of the incident wave. The wave reflected from the interface has the space frequency  $-K_1 \sin \theta_1$ .

A graph corresponding to the reflection must express the amplitude and phase changes occurring during reflection. It is convenient to represent the passage of the signal with transformation of the space frequency by a vertical arrow directed upwards (increase in frequency) or downward (decrease in frequency), and to represent the passage of a signal with preservation of the space frequency by a horizontal arrow. The amplitude of the incident wave is denoted by  $E$  and corresponds to the node 0. The amplitude of the reflected wave is denoted by  $E_r$  and denotes the node 2, where

$$E_r = E \cdot \Gamma_{21}, \quad (2)$$

where  $\Gamma_{21}$  is the coefficient of reflection from the interface 2-1.

When a wave passes through an interface its space frequency /6 remains unchanged, which is accounted for by the constancy of the coefficient of transmission through the interface. From the conservation of the space frequency of the wave when it passes through the interface there follows the law of refraction:

$$K_1 \sin \theta_1 = K_2 \sin \theta_2. \quad (3)$$

The amplitude of a refracted wave is denoted by  $E_x$  and corresponds to node 3, where

$$E_x = E \cdot \mathcal{X}_{12}, \quad (4)$$

where  $\mathcal{X}_{12}$  is the coefficient of transmission (radiation) of the wave through interface 1-2.

A graph describing the passage of a wave through an array of three media is shown in Fig. 2.

Here the branches  $\Gamma_{21}$ ,  $\Gamma_{12}$ , and  $\Gamma_{32}$  characterize the reflection of a wave at the boundaries of these media, and  $\mathcal{X}_{12}$ ,  $\mathcal{X}_{21}$ , and  $\mathcal{X}_{23}$  characterize the passage of the wave through the interfaces. The action of an intermediate medium II with thickness  $z_2$  is reflected by the horizontal arrow  $e^{-j\beta z_2}$

(  $\beta_n = K_n \cos \theta_n = \frac{2\pi}{\lambda} \sqrt{\epsilon_n} \cos \theta_n$  ), since in the passage of a wave through a homogeneous medium its space frequency remains unchanged.

Using Mason's algorithm [3] for the transfer coefficients of a graph, we can calculate the coefficient of transfer of this system for the reflected wave:

$$\Gamma' = \frac{\Gamma_{12}(1 - \Gamma_{22}\Gamma_{32}e^{-j\beta z_2}) + \mathcal{X}_{12}\mathcal{X}_{21}\Gamma_{32}e^{-j\beta z_2}}{1 - \Gamma_{22}\Gamma_{32}e^{-j\beta z_2}}. \quad (5)$$

The following relations are valid for the Fresnel coefficients:

$$\Gamma_{n,n+1} = -\Gamma_{n+1,n}; \quad \mathcal{X}_{n,n+1}\mathcal{X}_{n+1,n} - \Gamma_{n,n+1}\Gamma_{n+1,n} = 1. \quad (6)$$

Substituting (6) into (5), we get the coefficient of reflection of a three-layered structure: /7

$$\tilde{r}' = \frac{r'_{21} + r'_{32} e^{-j2\beta_2 z_2}}{1 + r'_{21} r'_{32} e^{-j2\beta_2 z_2}}. \quad (7)$$

An analogous expression is derived also by the usual method of calculating the amplitudes of the transmitted and the reflected waves.

A graph describing the passage [path] of a wave through a multilayered structure can be represented in the form shown in Fig. 3.

The coefficient of reflection of this multilayered structure can be obtained by means of Mason's algorithm and can be represented by the recursion formulas:

$$\begin{aligned} \tilde{r}'_{n-1,n-2} &= \frac{r'_{n-1,n-1} + r'_{n,n-1} e^{-j2\beta_{n-1} z_{n-1}}}{1 + r'_{n-1,n-1} r'_{n,n-1} e^{-j2\beta_{n-1} z_{n-1}}}, \\ \tilde{r}'_{n-2,n-3} &= \frac{r'_{n-2,n-2} + \tilde{r}'_{n-1,n-2} e^{-j2\beta_{n-2} z_{n-2}}}{1 + r'_{n-2,n-2} \tilde{r}'_{n-1,n-2} e^{-j2\beta_{n-2} z_{n-2}}}, \\ &\vdots \\ \tilde{r}'_{21} &= \frac{r'_{21} + \tilde{r}'_{32} e^{-j2\beta_2 z_2}}{1 + r'_{21} \tilde{r}'_{32} e^{-j2\beta_2 z_2}}. \end{aligned} \quad (8)$$

The emissivity of this multilayered structure is

$$\mathcal{E}_{\lambda 1} = 1 - (\tilde{r}'_{21})^2. \quad (9)$$

Here  $\tilde{r}'_{n-1,n-2}, \dots, r'_{n,n-1}$ , and so on are the Fresnel coefficients:

$$\tilde{r}'_{n,n-1} \text{ horizontal} = \frac{\sqrt{\epsilon_n} \cos \theta_n - \sqrt{\epsilon_{n-1}} \cos \theta_{n-1}}{\sqrt{\epsilon_n} \cos \theta_n + \sqrt{\epsilon_{n-1}} \cos \theta_{n-1}}. \quad (10)$$

$$\tilde{r}'_{n,n-1} \text{ vertical} = \frac{\sqrt{\epsilon_n} \cos \theta_n - \sqrt{\epsilon_{n-1}} \cos \theta_{n-1}}{\sqrt{\epsilon_{n-1}} \cos \theta_n + \sqrt{\epsilon_n} \cos \theta_{n-1}}. \quad (11)$$

This method permits determining any functions of the emissivity of a multilayered structure with any number of layers:



the polarization characteristics  $\mathcal{I}_{\text{hor}}(\theta)$  and  $\mathcal{I}_{\text{vert}}(\theta)$  for variations in  $n$ ,  $\lambda$ ,  $\epsilon$ , and  $z$ ; the dependence of emissivity on wavelength  $\mathcal{E}(\lambda)_{z, \epsilon, \eta}$ ; and the dependence of emissivity on layer thickness  $\mathcal{E}(z)_{\lambda, \epsilon, \eta}$ , and so on. However, in this form, the method does not permit allowing for the temperature profile. In these calculations it was assumed that all layers are at the same thermodynamic temperature. Further development of the method is needed to make allowance for the temperature profile.

## 2. Investigation of Radio Emission of Several Complex Formations of the Snow-Ice-Water Type

The proposed method permitting a relatively rapid calculation based on the recursion formulas (8,9,10,11) on a Naira electronic computer of the following structures, represented in the corresponding graphs: air-snow-ice; air-ice-water; water film on ice (air-water-ice); and air-snow-ice-water.

Fig. 4 presents the coefficient of radiation of the ice-water and ice-ground structures (the electrical parameters are indicated in the figures) as a function of the thickness of the ice layer for wavelengths 3 cm and 18 cm for observations at the nadir. The interference fluctuations in the emissivity of the layered structure are clearly observable, where the substrate (ground and water) has virtually no effect on the period of fluctuations, this being determined mainly by the electrical properties of the ice layer and the radiation wavelength. We note that a thin ice of ice several millimeters thick (approximately 3-5 mm) can sharply modify the radiation characteristics of the medium, both water as well as ice -- an addition to the brightness temperatures for water is represented by a value exceeding 100° K, and of more than 30° K for ground (when the thermodynamic temperature of the medium is 300° K). And, in the case of the ice-water structure, its brightness temperature is higher than the brightness temperature of water (the structure is "warmer" than the substrate); but in the

case of the ice-ground structure, the brightness temperature at the minimum can be lower than the ground temperature (the structure is "colder" than the substrate).

From an analysis of the polarization characteristics of the ice-water structures (wavelength 3 cm) shown in Figs 5 and 6, it follows that the strongest effect of the ice cover is seen in the horizontal component of the emissivity -- as a function of the dependence of this constituent on the angle for  $z/\lambda > 1$  pseudo-Brewster's angles appear, which are absent for an open water surface.

Fig. 7 presents the results of calculating the emissivity of the air-water-ice structure (water film on ice) as a function of wavelength for various thicknesses of the water film, varying from 0.1 cm to 3 cm, where the angle of observation  $\theta = 0^\circ$  and  $S_{\text{water}} = 0$  per mil.

In the 10-40 cm wavelength range, for water film thicknesses of 0.3-3 cm, interference phenomena are observed. It should be noted that a thin layer of water can appreciably reduce the brightness temperature of the structure ("cool" it); for example, at the wavelength 10 cm, the contrast between the brightness temperatures of the water-ice and ice structures is more than  $150^\circ$  K.

Fig. 8 gives the dependence of the emissivity of this same structure on the wavelength for a water film with thickness  $z = 0.1$  cm and  $z = 0.5$  cm, but for different electrical characteristics of the media. As to be expected, the dielectrical characteristics of water in this case play a key role and change both the value of the emissivity as well as the shape of the frequency functions, this effect being intensified with increase in wavelength. A substantial change in the electrical losses of ice ( $\text{tg } \delta = 0-0.5$ , which corresponds to sea ice) weakly affects the radiation characteristics of the structure. /10

Let us examine the following air-snow-ice structure. Fig. 9 presents the curves of the emissivity of this structure as a

function of the thickness of the snow layer for different wavelengths  $\lambda = 3, 18, \text{ and } 75$  for an angle of observation  $\theta = 0^\circ$  without allowing for losses in the media. The presence of the snow layer increases the brightness temperature of the structure, all the way to the emissivity values of unity (an absolute black body). Fig. 10 presents the frequency functions of the emissivity  $\epsilon$  of this structure for snow thicknesses  $z = 10 \text{ cm}$  and  $z = 70 \text{ cm}$ , where the angle of observation  $\theta = 0^\circ$ , the losses in the media not being taken into account.

Shown in Figs. 11 and 12 are the polarization characteristics of this structure for  $\lambda = 3 \text{ cm}$  with and without allowing for the losses in snow and ice and for various snow thicknesses  $z = 3 \text{ cm}$  and  $z = 10 \text{ cm}$ . Here several points are of interest: for  $z/\lambda > 1$  interference phenomena are observed -- (pseudo-Brewster's angles); the functions  $X_h(\theta)$ ,  $X_v(\theta)$ , and  $P(\theta)$  for  $z/\lambda > 1$  differ sharply from the corresponding functions for  $z/\lambda < 1$ . The largest losses in the snow ( $\epsilon''_{\text{snow}} = 0.05$ ) smooth out this difference (Fig. 11) and are strongly reflected in the dependence of the coefficient of polarization on the angle of observation (Fig. 12) (absence of zeroes at the angles  $40-70^\circ$ ). This decrease in the amplitude of the oscillations of the radiation coefficient and the coefficient of polarization is due to an increase in the absorption of the electromagnetic energy of the overall flux of radiation in the snow layer.

Allowing for losses in the ice ( $\epsilon''_{\text{ice}} = 0-0.05$ ) has practically no effect on the polarization characteristics of this structure. /11 From these graphs it can be concluded that in the range  $\lambda = 3 \text{ cm}$  snow thickness can be determined from the polarization characteristics.

Let us examine the following air-snow-ice-water structure. Given in Figs. 13 and 14 are the polarization characteristics of this structure in the range  $\lambda = 3 \text{ cm}$  for different ice thicknesses  $z_{\text{ice}} = 15 \text{ cm}$  (Fig. 13) and  $z_{\text{ice}} = 50 \text{ cm}$  (Fig. 14) and for the snow

thickness  $z_{\text{snow}} = 10$  cm, without allowing for losses in the snow and ice. The presence of a water substrate is very strongly reflected in the polarization characteristics of this structure.

Fig. 15 gives the polarization characteristics of this structure for a snow thickness  $z_{\text{snow}} = 10$  cm and an ice thickness  $z_{\text{ice}} = 5$  m and for different losses in ice and snow; the salinity of the water  $S = 0$  per mil,  $t = 0^\circ$  C. For this ice thickness, the presence of a water substrate has virtually no effect on the polarization characteristics of the structure (compare Figs. 15 and 11).

The outlays of machine time for calculating this structure (Figs. 13, 14, 15) on a Naira electronic computer were of the order of 2 hours.

### Conclusions

The method of calculating multilayered structures with the aid of oriented graphs using a Naira electronic computer makes it possible with small outlays of machine time to analyze the radio emission of complex layered structures with any number of layers with different thickness and with arbitrary electrical parameters.

Further elaboration of the method will permit allowing for the temperature profile of the structure and calculating multilayered structures with random array of layers and their dielectrical characteristics.

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/12

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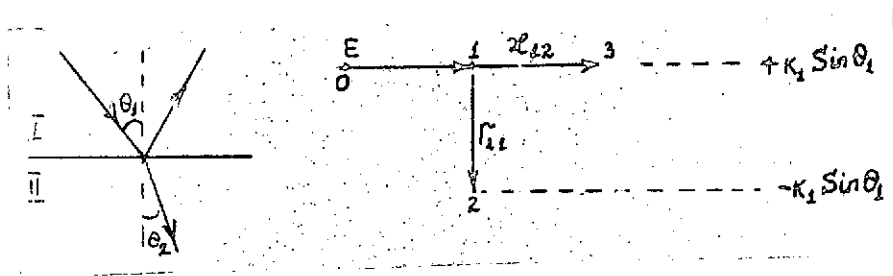


Fig. 1. Plot of path of wave through interface of two media

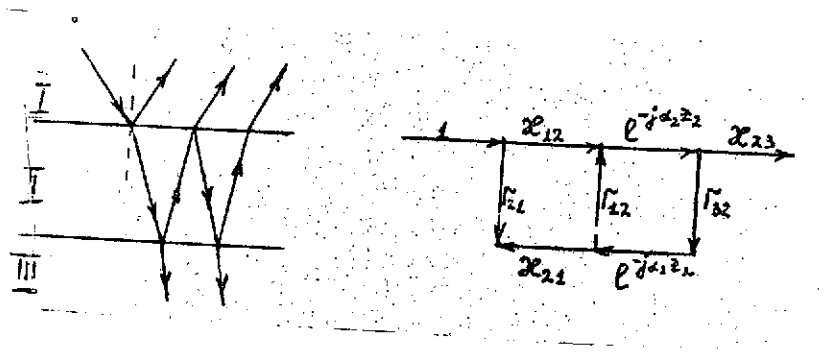


Fig. 2. Plot of path of wave through array of three media

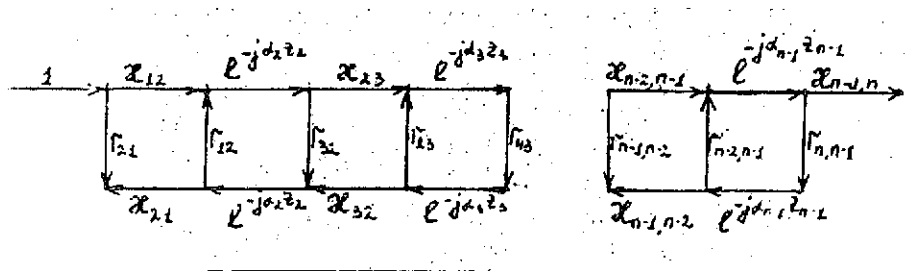


Fig. 3. Plot of path of wave through multilayer structure

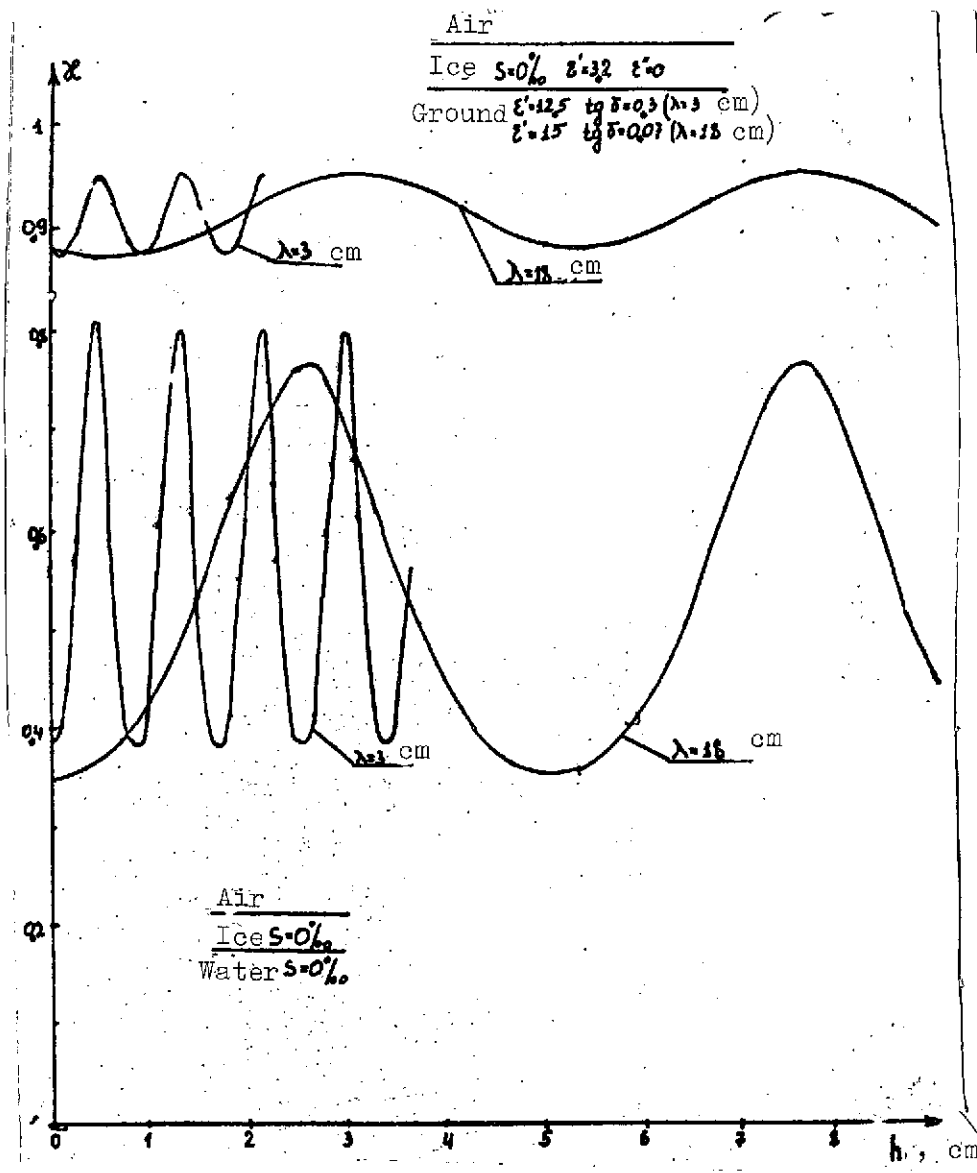


Fig. 4. Dependence of coefficient of radiation ice-water and ice-ground structure on thickness of ice layer

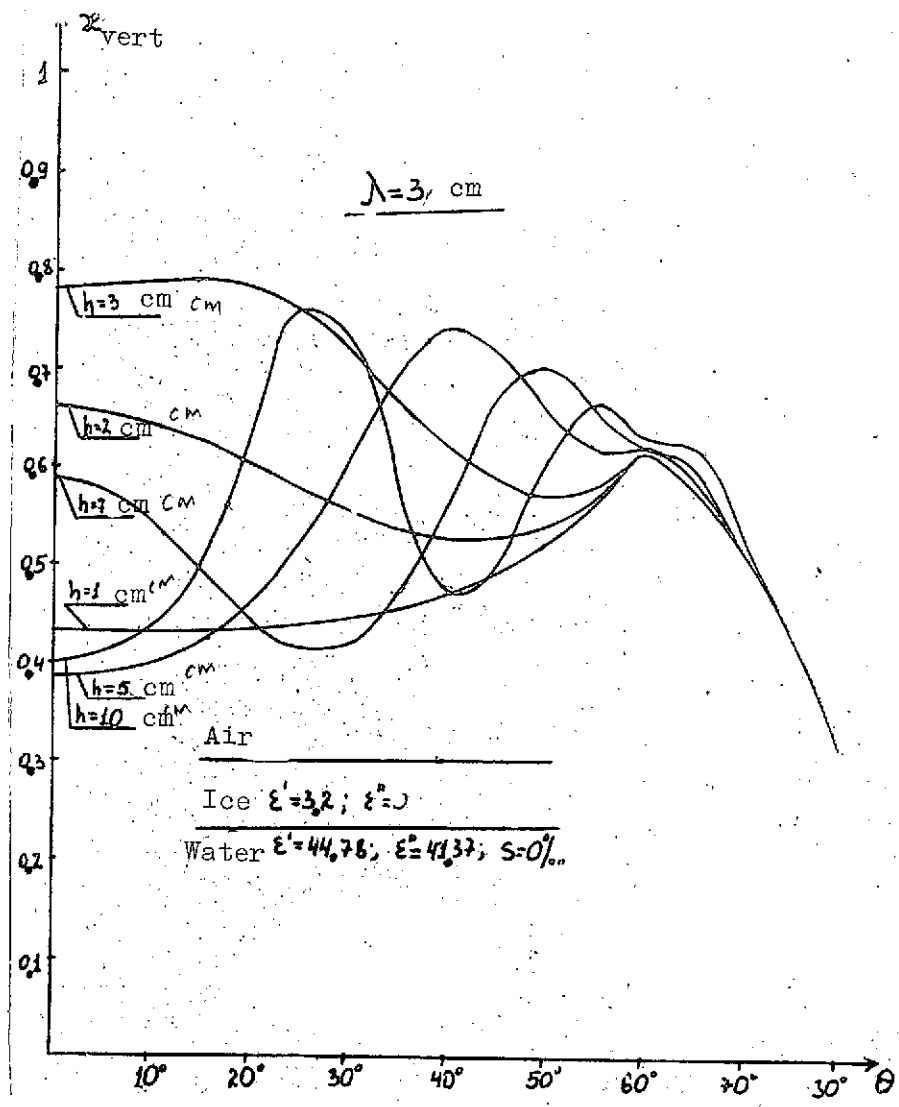


Fig. 5. Polarization characteristics of ice-water structures



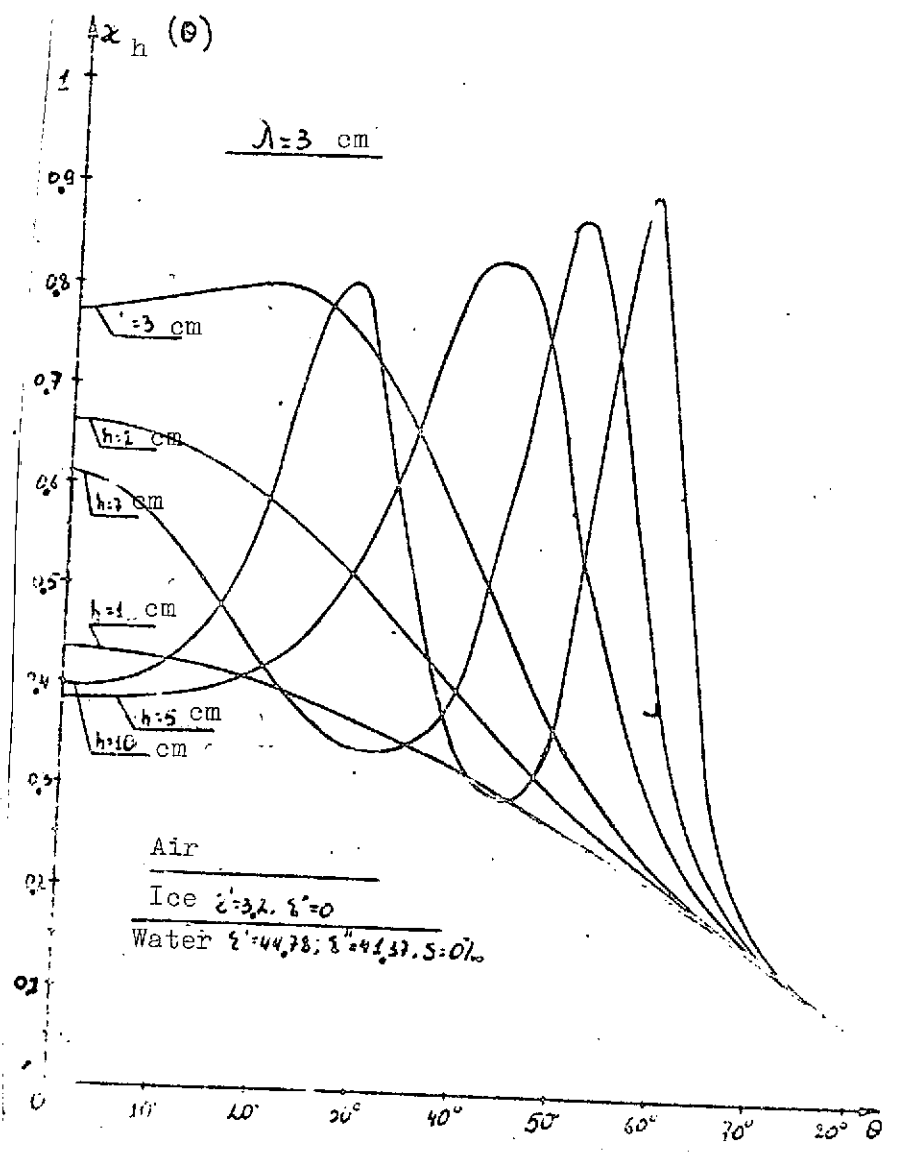


Fig. 6. Polarization characteristics of ice-water structures

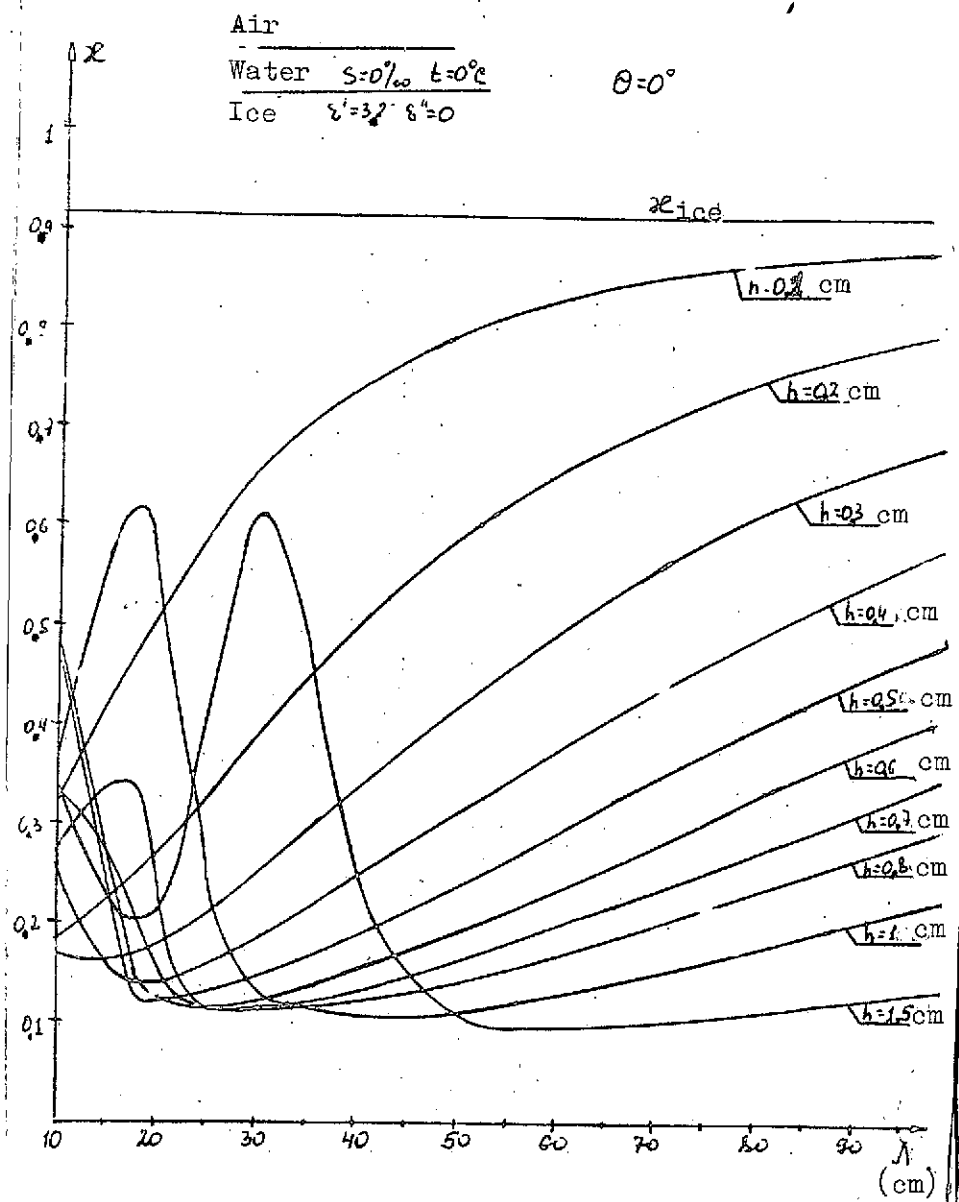


Fig. 7. Frequency characteristics of structure of water film on ice for various film thicknesses

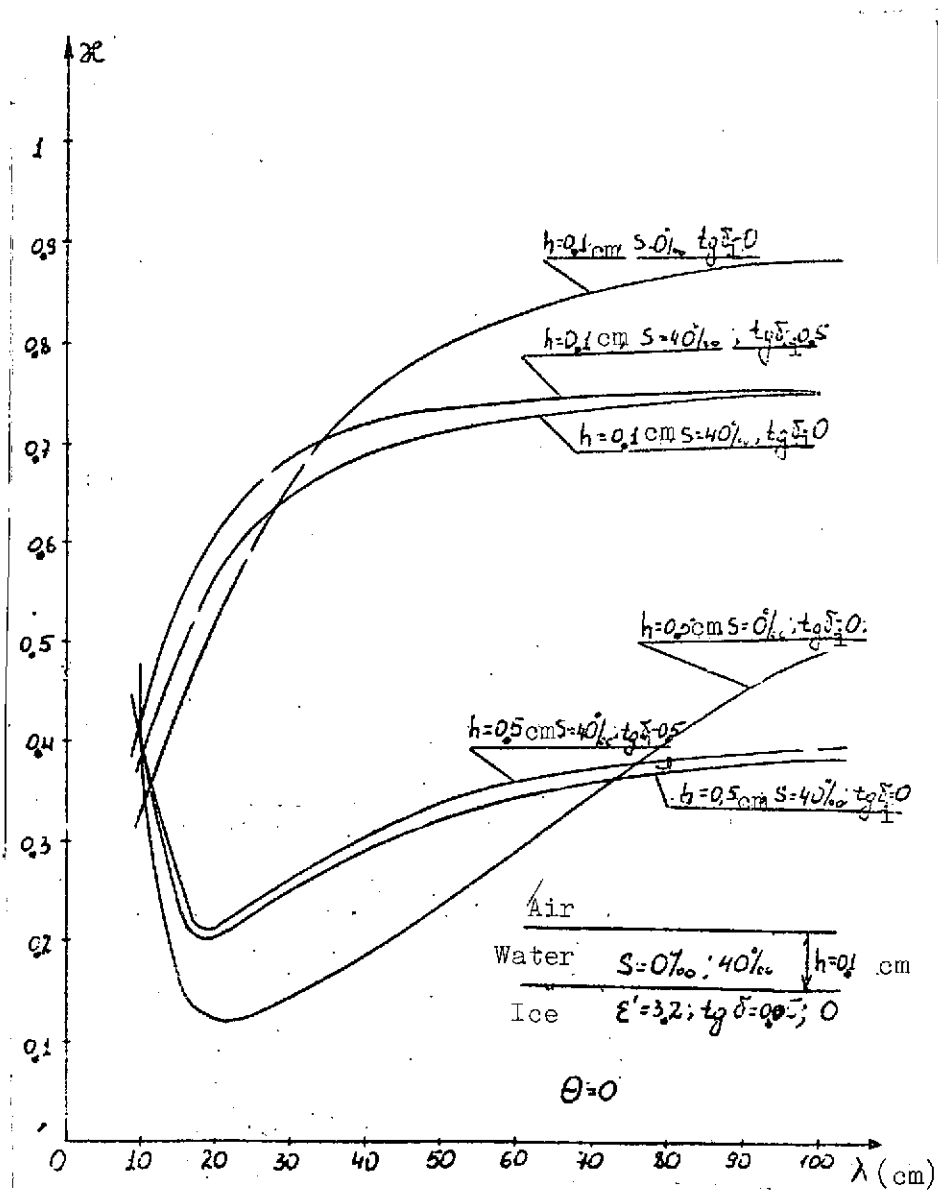


Fig. 8. Frequency characteristics of structure of water film on ice for various film thicknesses

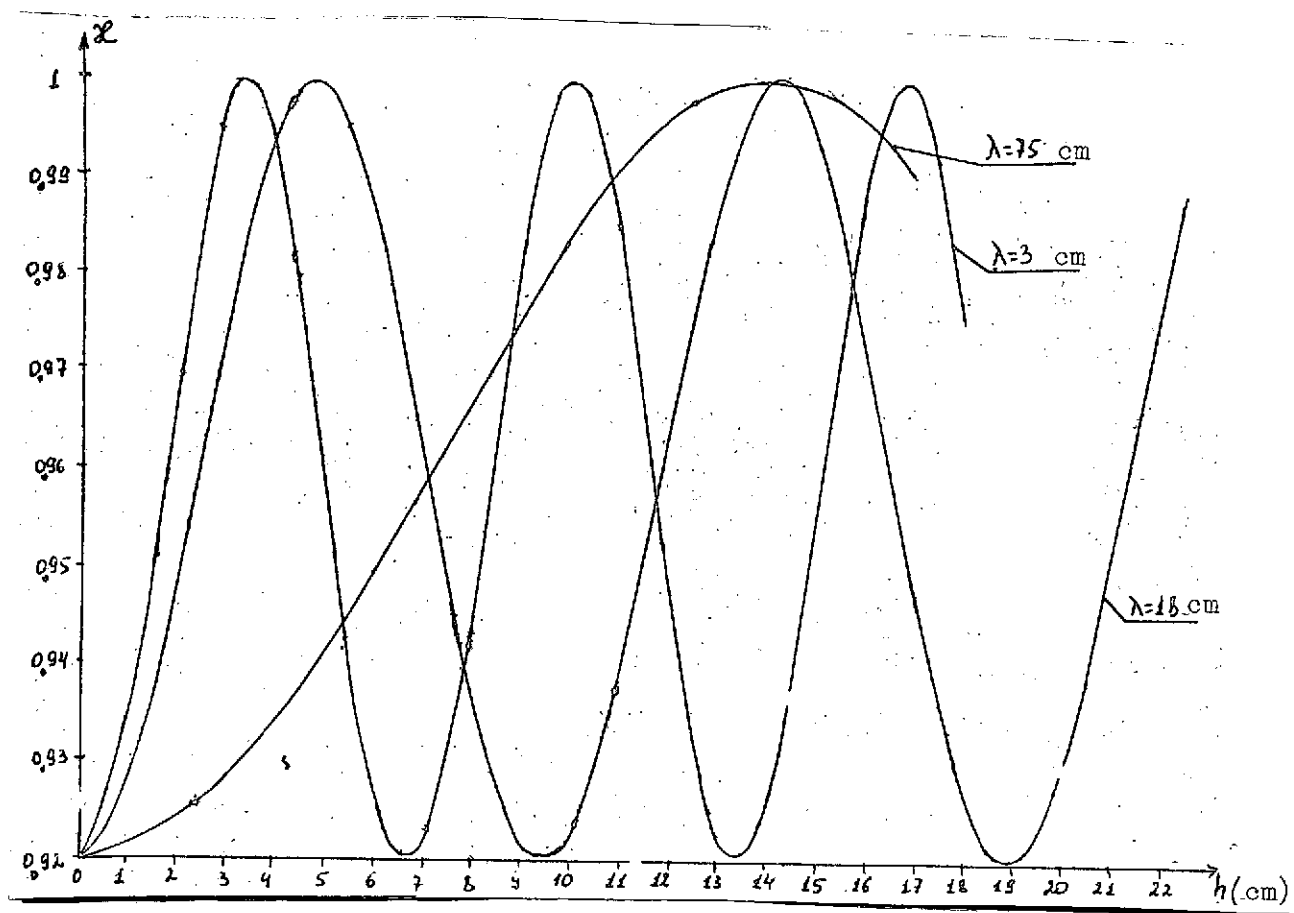


Fig. 9. Dependence of emissivity of snow-ice structure on thickness of snow layer for various wavelengths

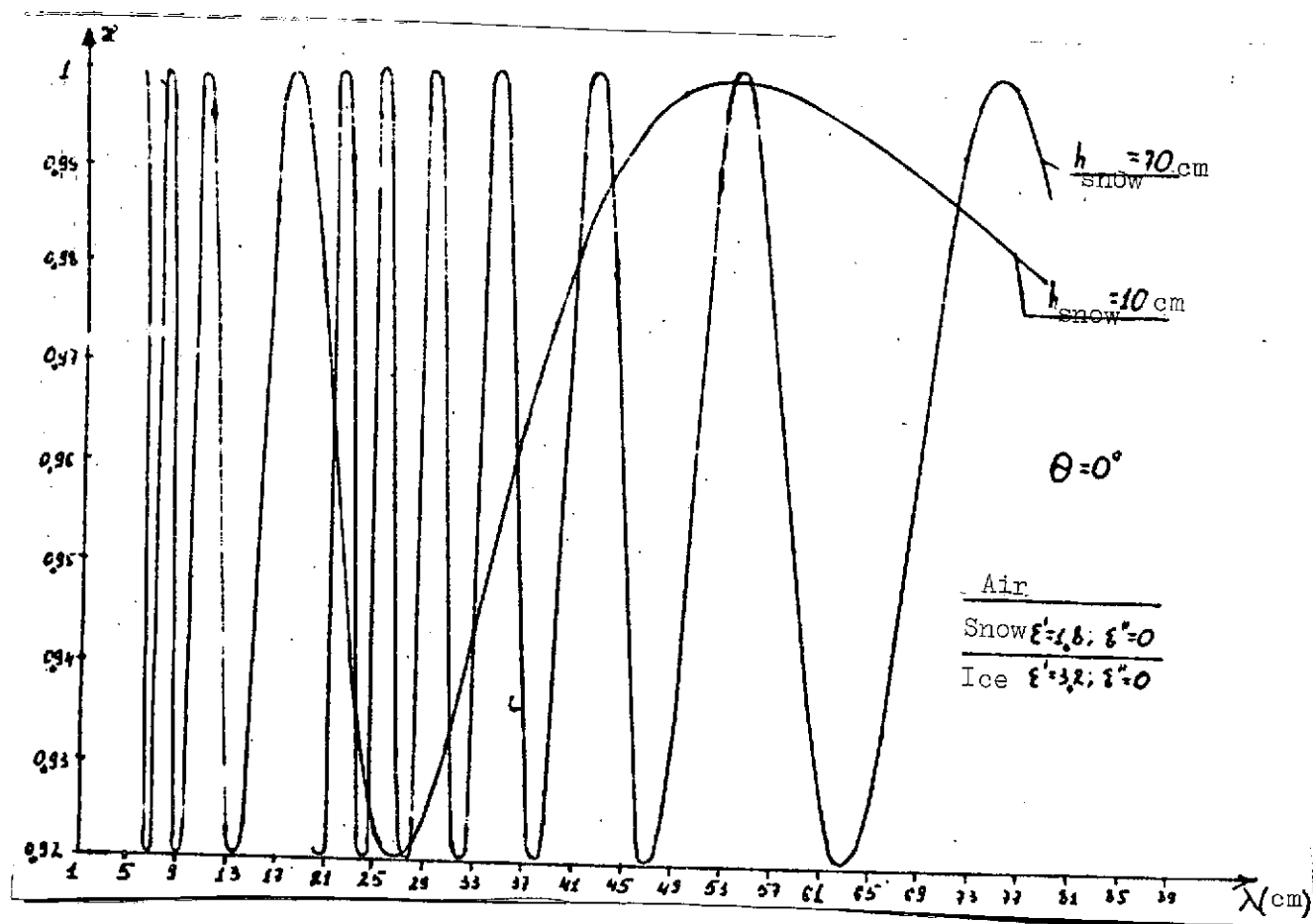


Fig. 10. Frequency characteristics of snow-ice structures for various snow thicknesses

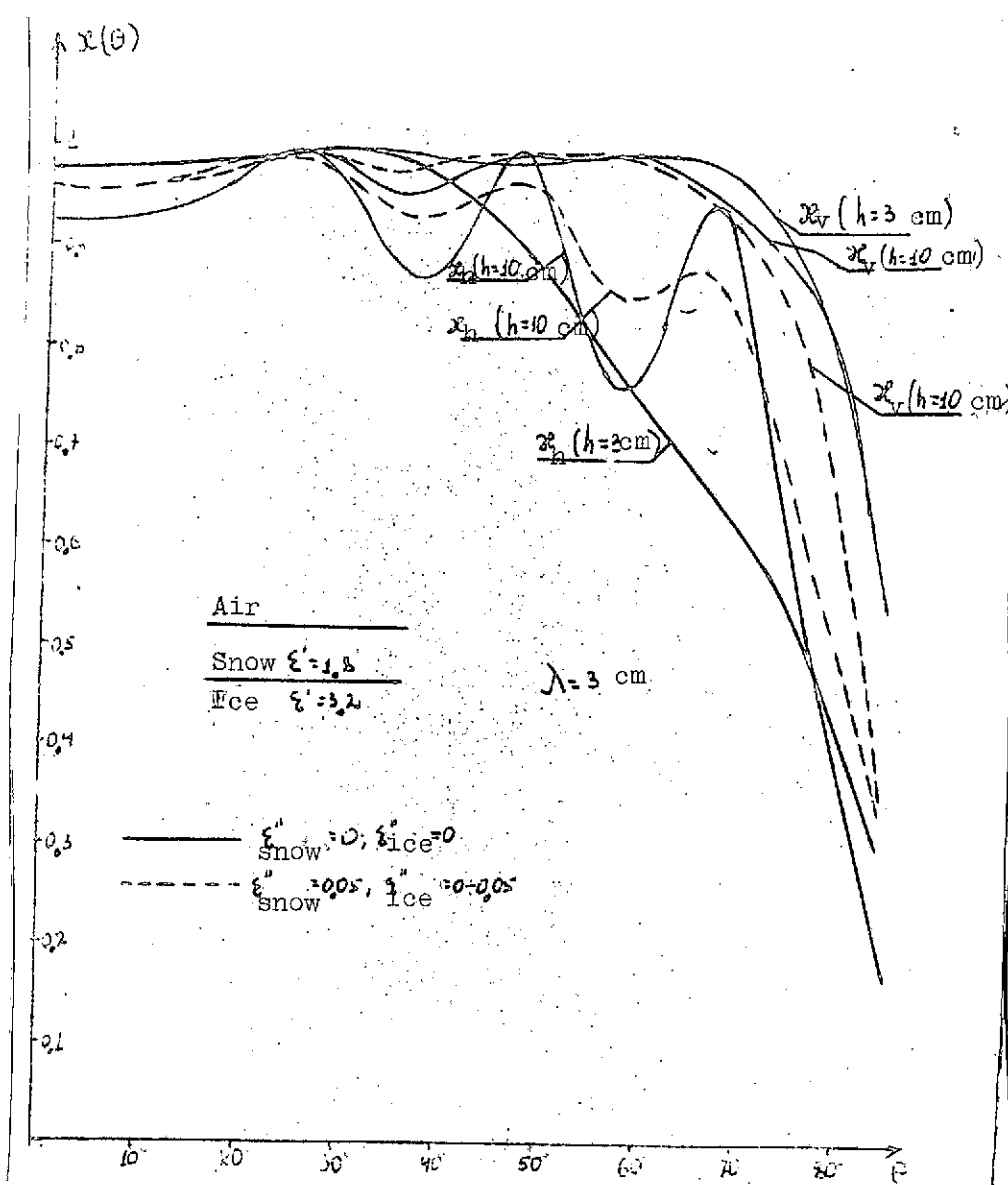


Fig. 11. Polarization characteristics of snow-ice structure with and without allowance for losses in snow and ice

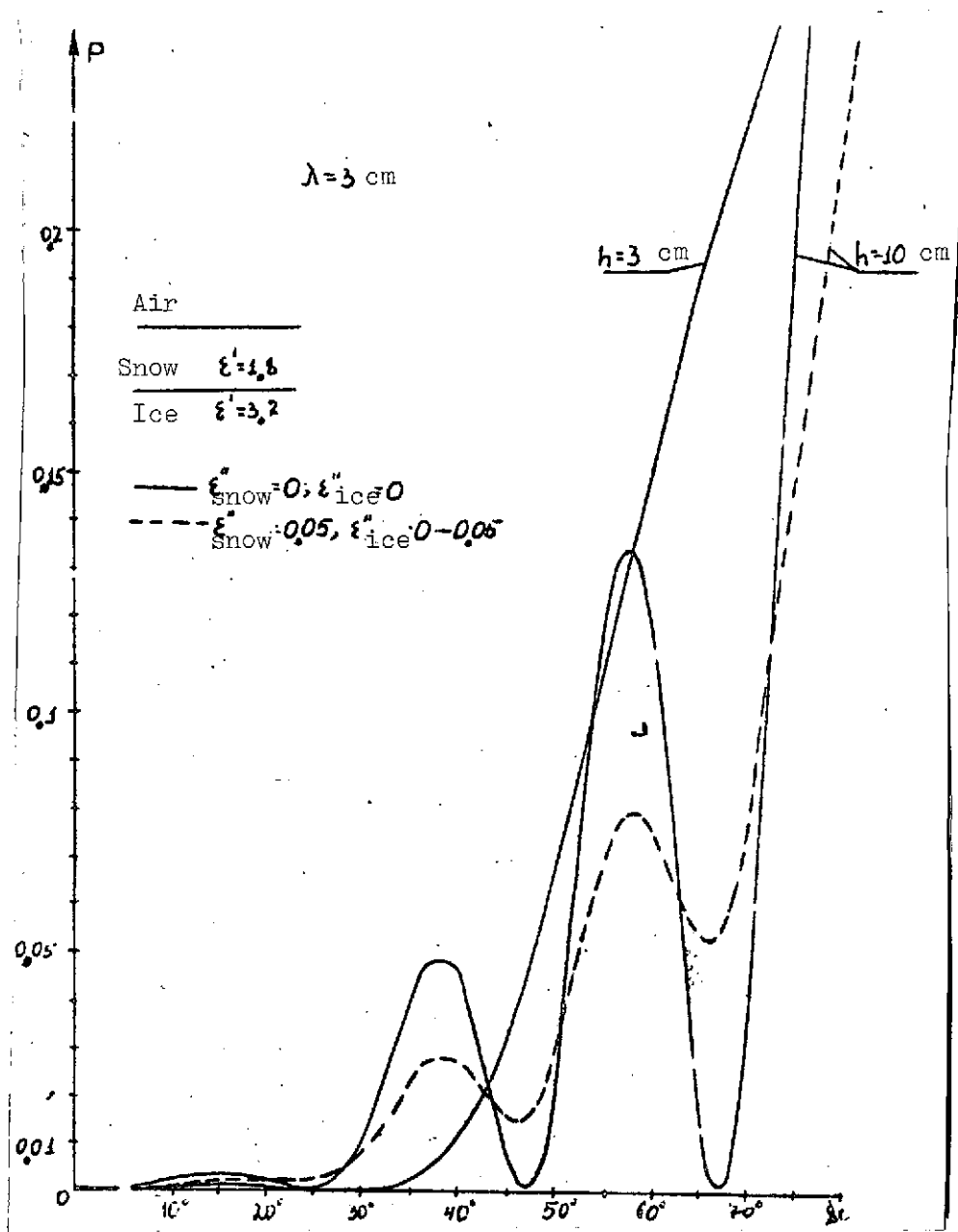


Fig. 12. Polarization characteristics of snow-ice structure with and without allowance for losses in snow and ice

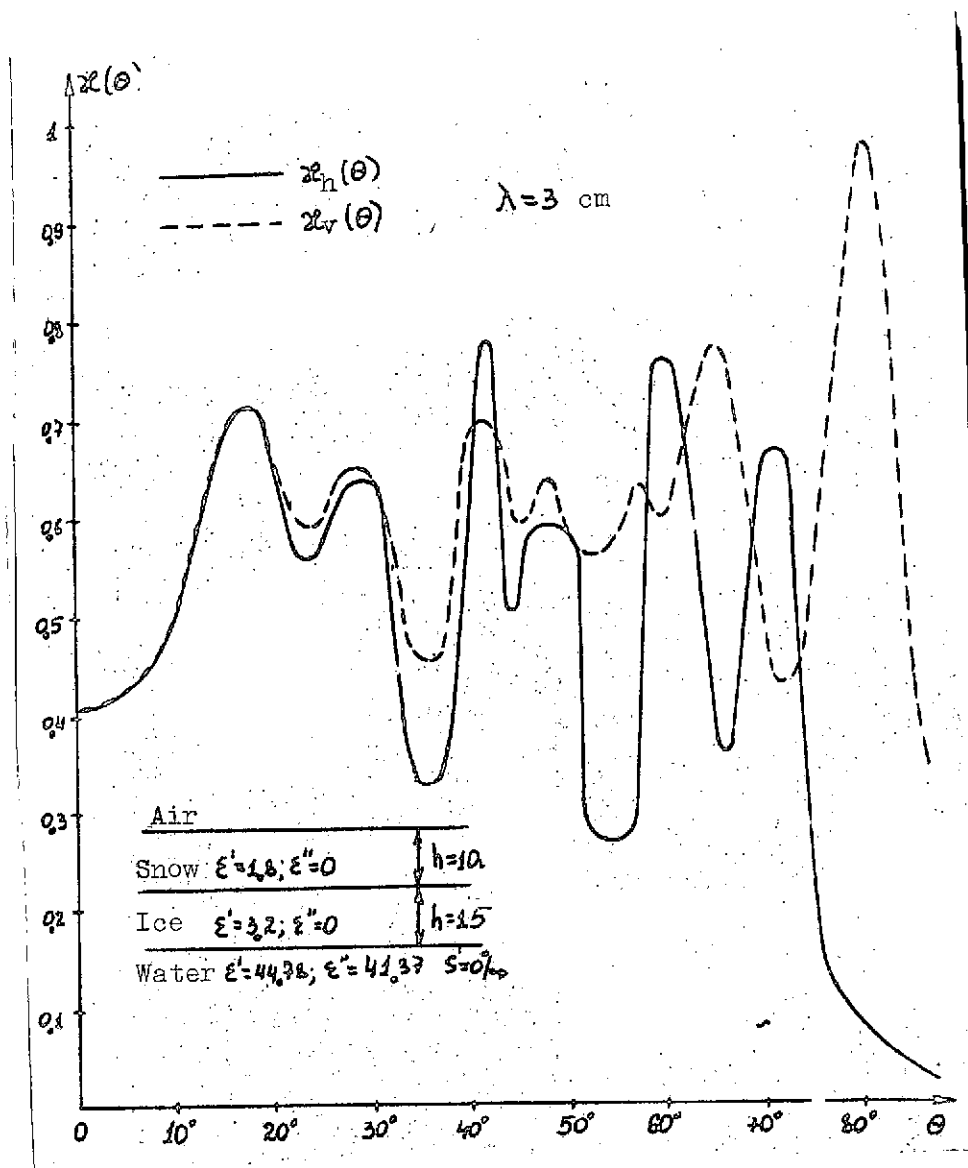


Fig. 13. Polarization characteristics of snow-ice-water structures for various thicknesses of snow and ice with and without allowance for losses in snow and ice



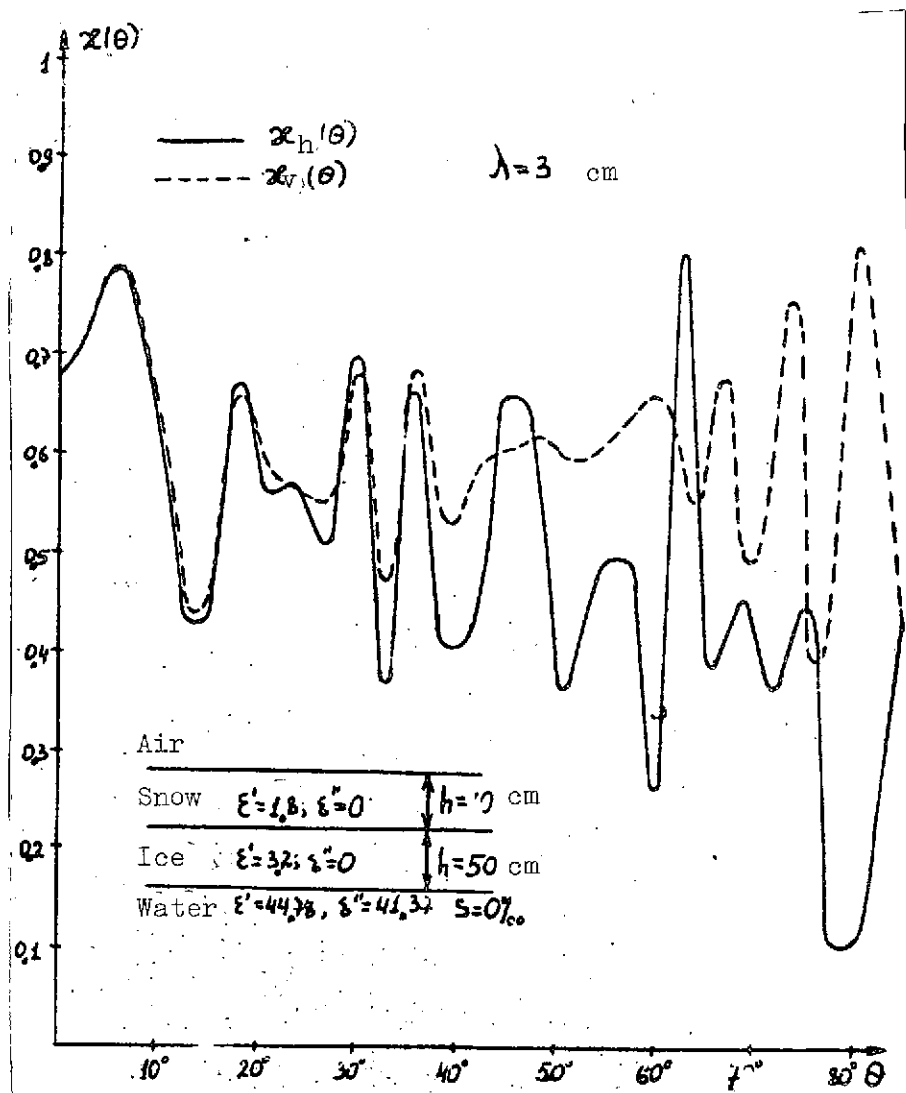


Fig. 14. Polarization characteristics of snow-ice-water structures for various thicknesses of snow and ice with and without allowance for losses in snow and ice

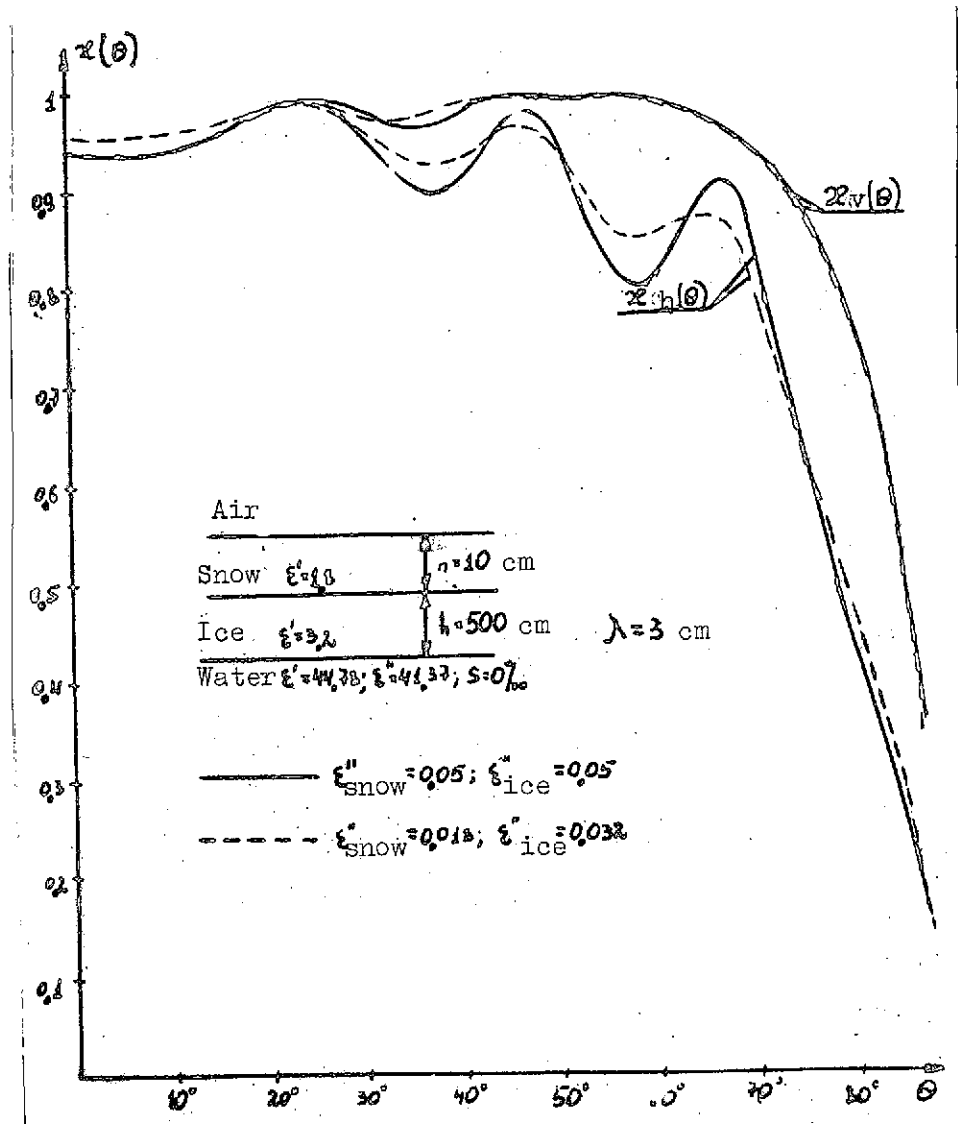


Fig. 15. Polarization characteristics of snow-ice-water structures for various thicknesses of snow and ice with and without allowance for losses in snow and ice